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FACULTY OF ENGINEERING (SHOUBRA)
ELECTRONICS AND COMMUNICATIONS ENGINEERING

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FACULTY OF ENGINEERING AT SHOUBRA

ECE 444
Industrial Electronics

(2022 - 2023) 1st term

Lecture 7: Controller Principles (part2).

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Outlines:

- Continuous Controller Modes.
- Composite Control Modes.

Continuous Controller Modes:
1- Proportional Control Mode

- Smooth linear relationship exists between the controller output & the error.

$$P = Kp.e_p + P_0$$

K_P = proportional gain between error and controller output (% per %).
P_0 = controller output with no error (%).

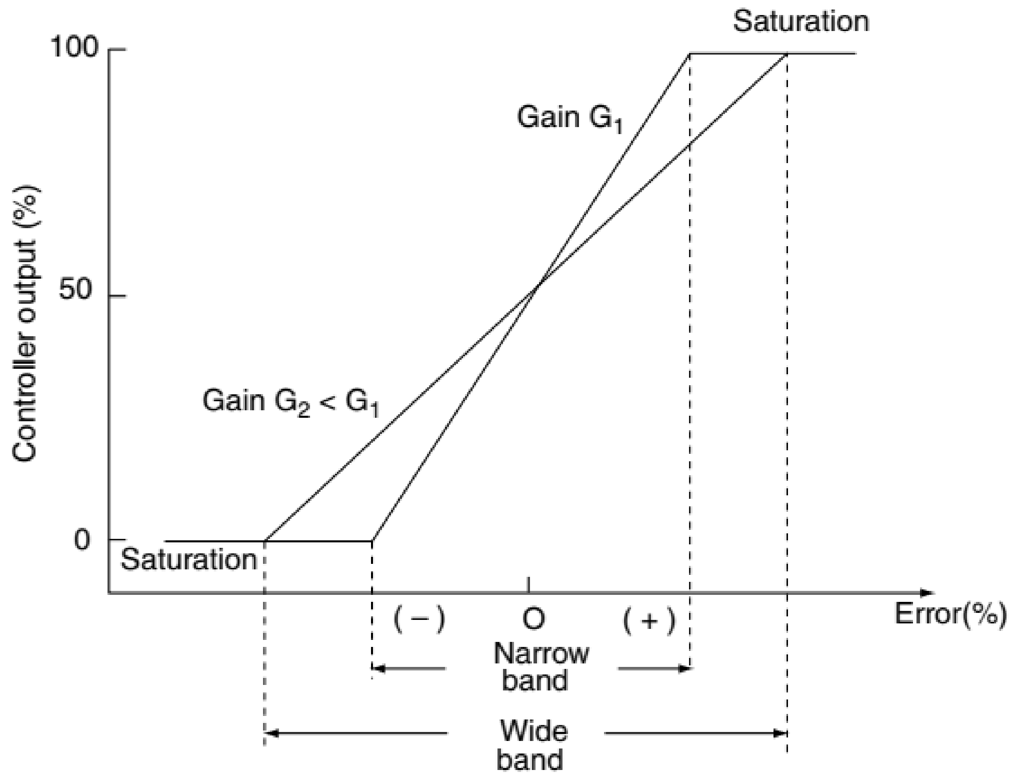
Direct action: $P = -Kp.e_p + P_0$

Reverse action: $P = Kp.e_p + P_0$

- Proportional band (PB): the range of the error to cover the 0% to 100% of the controller output.

$$PB = \frac{100}{K_P}$$

- A high gain means large response to an error (fast), but also a narrow error band within which the output is not saturated.
- A low gain means slow response to an error, but also a wide error band.

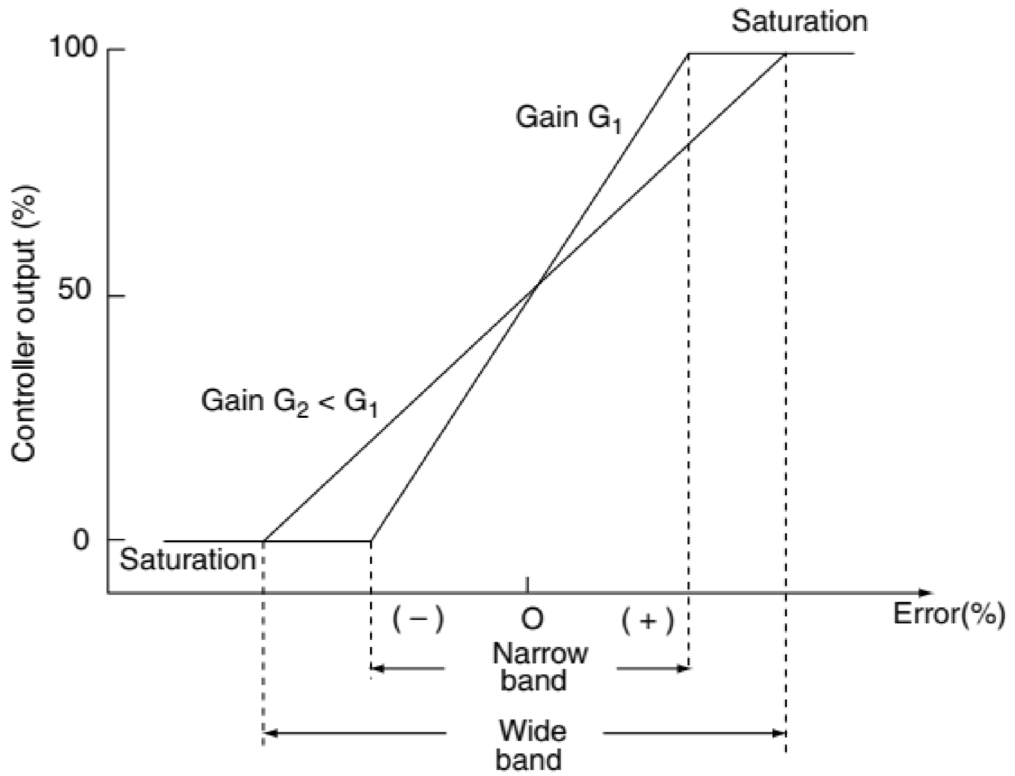


Continuous Controller Modes:
1- Proportional Control Mode

- Let us summarize the characteristics of the proportional mode:
  1. If the error is zero, the output is a constant equal to P_0 .
  2. If there is error, for every 1% of error, a correction of K_P percent is added to or subtracted from P_0 , depending on the sign of the error.
  3. There is a band of error about zero of magnitude PB within which the output is not saturated at 0% or 100%.

- **Offset:** An important characteristic of the proportional control mode is that it produces a permanent steady state error in the operating point of the controlled variable when a change in load occurs. This error is referred to as steady state offset.
- It can be minimized by a larger constant, K_P , which also reduces the proportional band. (on/off control !)

$$e_p = \frac{p - p_0}{K_P}$$



Continuous Controller Modes:
1- Proportional Control Mode

EXAMPLE 6

Consider the proportional-mode level-control system of Figure 13. Value A is linear, with a flow scale factor of 10 m^3/h per percent controller output. The controller output is nominally 50% with a constant of $K_P = 10\%$ per %. A load change occurs when flow through valve B changes from 500 m^3/h to 600 m^3/h. Calculate the new controller output and offset error.

Solution

Certainly, valve A must move to a new position of 600 m^3/h flow or the tank will empty. This can be accomplished by a 60% new controller output because

$$Q_A = \left(\frac{10\ \text{m}^3/\text{h}}{\%}\right)(60\%) = 600\ \text{m}^3/\text{h}$$

as required. Because this is a proportional controller, we have

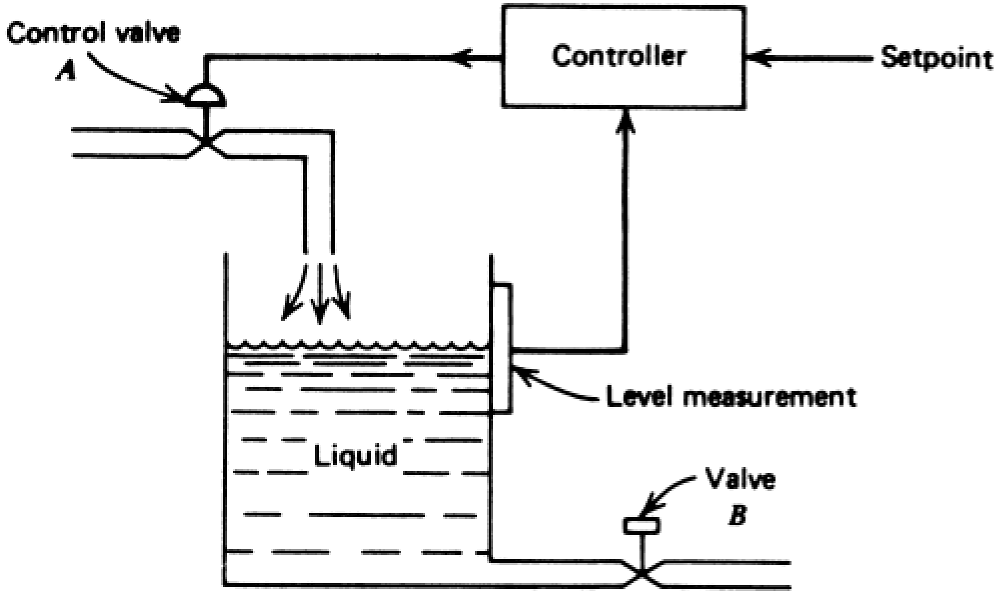
$$p = K_P e_p + p_0$$

with the nominal condition $p_0 = 50\%$. Thus

$$e_p = \frac{p - p_0}{K_P} = \frac{60 - 50}{10}\%$$

$$e_p = 1\%$$

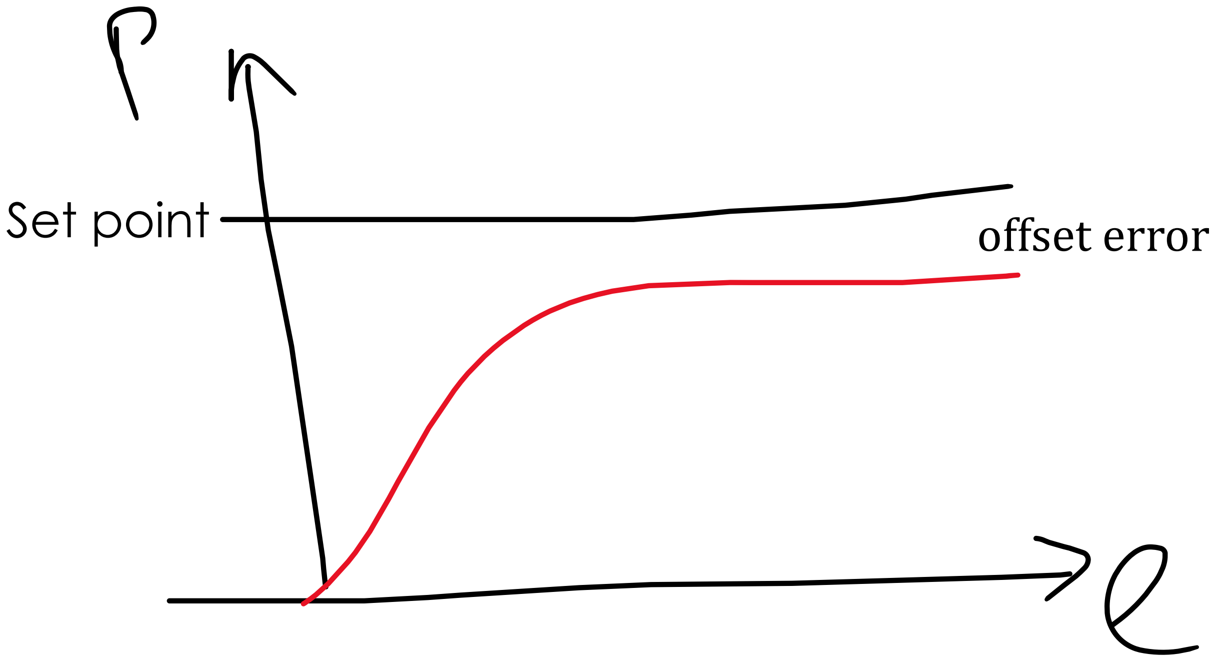
so a 1% offset error occurred because of the load change.



Continuous Controller Modes:
1- Proportional Control Mode

- Adv.: No oscillation.
- Dis. Adv.: Steady state error.
- Application: The offset error limits use of the proportional mode to only a few cases, particularly those where a manual reset of the operating point is possible to eliminate offset.(manual change of P_0)

- K_P trade off!?

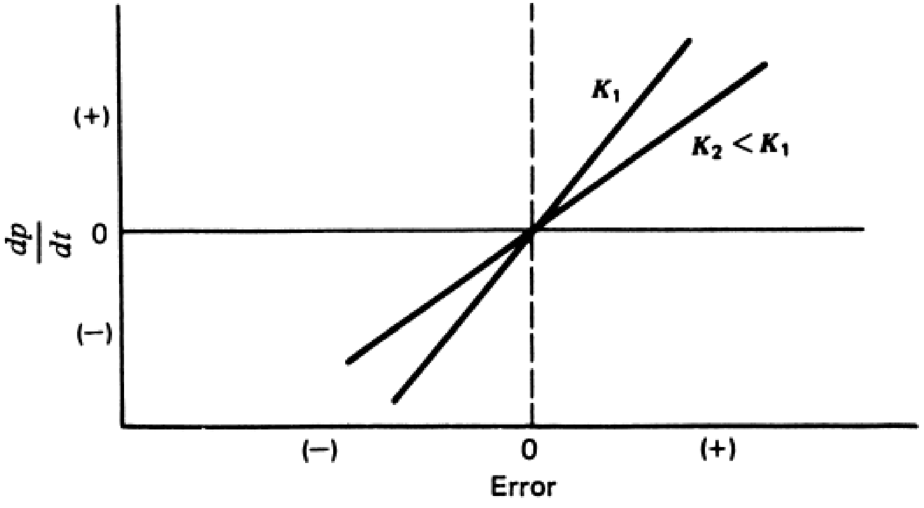


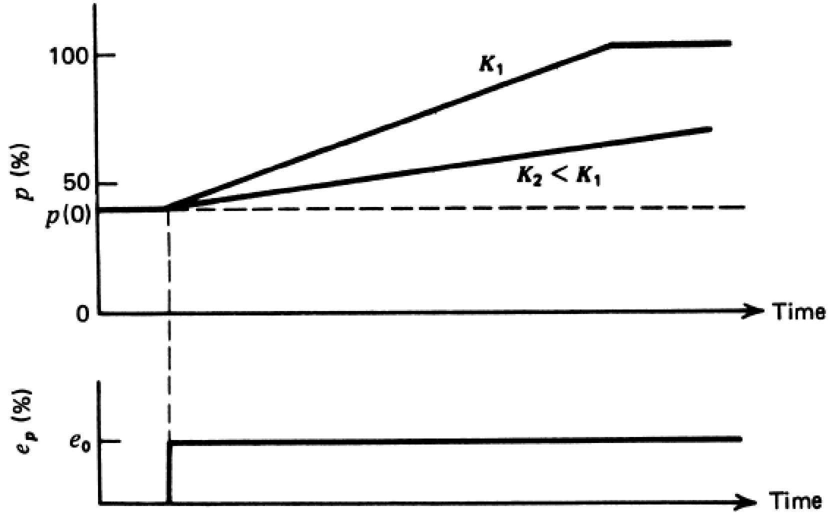
Continuous Controller Modes:
2- Integral Control Mode (Reset Action)

- The integral mode eliminates the offset error by allowing the controller to adapt to changing external conditions by changing the zero-error output.
- Integral action is provided by summing the error over time, multiplying that sum by a gain, and adding the result to the present controller output.

$$\frac{dp}{dt} = K_I e_p$$

$$p(t) = K_I \int_0^t e_p \, dt + p(0)$$





- Where: Ki: integral gain (sec^{-1})
  Ti: integral (reset) time =1/Ki (sec.)
  P(0): the controller output when the integral action starts.
- If the error is zero, the controller output is not changed.
- If the error is not zero, the output will begin to increase or decrease at a rate of K_I percent/second for every 1% of error

Continuous Controller Modes:
2- Integral Control Mode (Reset Action)

EXAMPLE 7 An integral controller is used for speed control with a setpoint of 12 rpm within a range of 10 to 15 rpm. The controller output is 22% initially. The constant $K_I = -0.15\%$ controller output per second per percentage error. If the speed jumps to 13.5 rpm, calculate the controller output after 2 s for a constant e_p.

Solution

We find e_p from Equation (3):

$$e_p = \frac{r - b}{b_{max} - b_{min}} \times 100$$

$$e_p = \frac{12 - 13.5}{15 - 10} \times 100$$

$$e_p = -30\%$$

The rate of controller output change is then given by Equation (17),

$$\frac{dp}{dt} = K_I e_p = (-0.15\ s^{-1})(-30\%)$$

$$\frac{dp}{dt} = 4.5\%/s$$

The controller output for constant error will be found from Equation (16)

$$p = K_I \int_0^t e_p\, dt + p(0)$$

but because e_p is constant,

$$p = K_I e_p t + p(0)$$

After 2 s, we have

$$p = (0.15)(30\%)(2) + 22$$

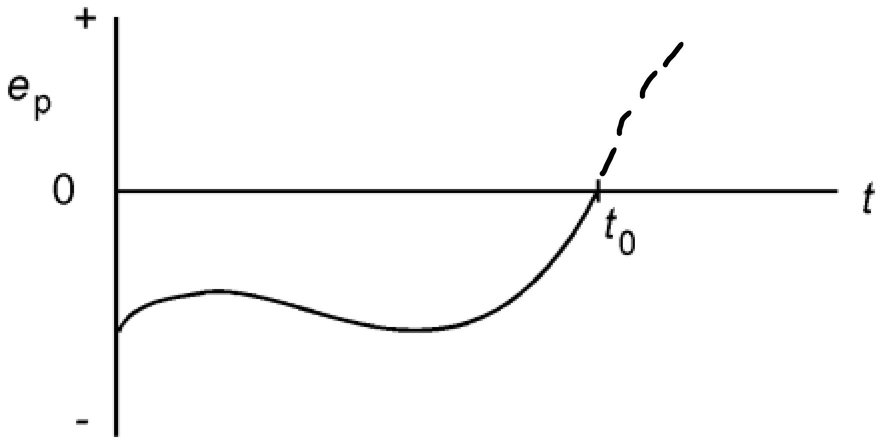
$$p = \mathbf{31\%}$$

Continuous Controller Modes:
2- Integral Control Mode (Reset Action)

- Adv.: No Steady state error.
- Application: Typically, the integral mode is not used alone, but can be used for systems with small process lags and correspondingly small capacities. (saturation)

Continuous Controller Modes:
2- Derivative Control Mode (Rate Action-Anticipatory Action)

- Suppose you were in charge of controlling some variable, and at some time, , your helper yelled out, “The error is zero. What action do you want to take?” Well, it would seem perfectly rational to answer “None” because, after all, the error was zero.
- But suppose you have a screen that shows the variation of error in time and that it looks like Figure.
- You can clearly see that even though the error at is zero, it is changing in time and will certainly not be zero in the following time. Therefore, some action should be taken even though the error is zero!
- Derivation controller action responds to the rate at which the error is changing that is, the derivative of the error.



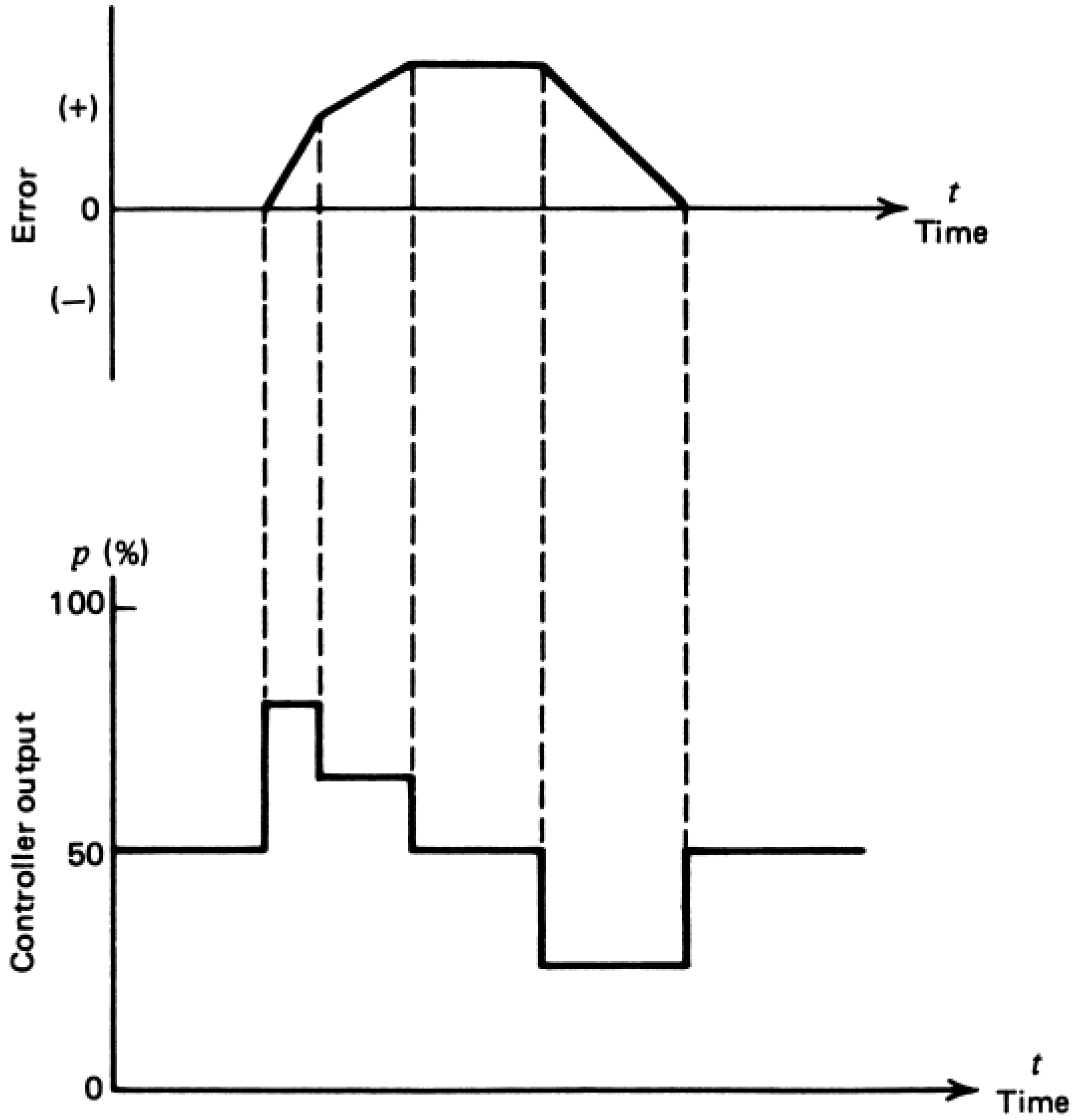
$$p(t) = K_D \frac{de_p}{dt}$$

- Where K_D: Rate or derivative time (sec).
- **Can't be used alone.. why?**

Continuous Controller Modes:
2- Derivative Control Mode (Rate Action-Anticipatory Action)

1. If the error is zero, the mode provides no output.
2. If the error is constant in time, the mode provides no output.
3. If the error is changing in time, the mode contributes an output of percent for every 1%-per-second rate of change of error.

- Adv.: Fast response.
- Dis.adv.: Can't be used alone.



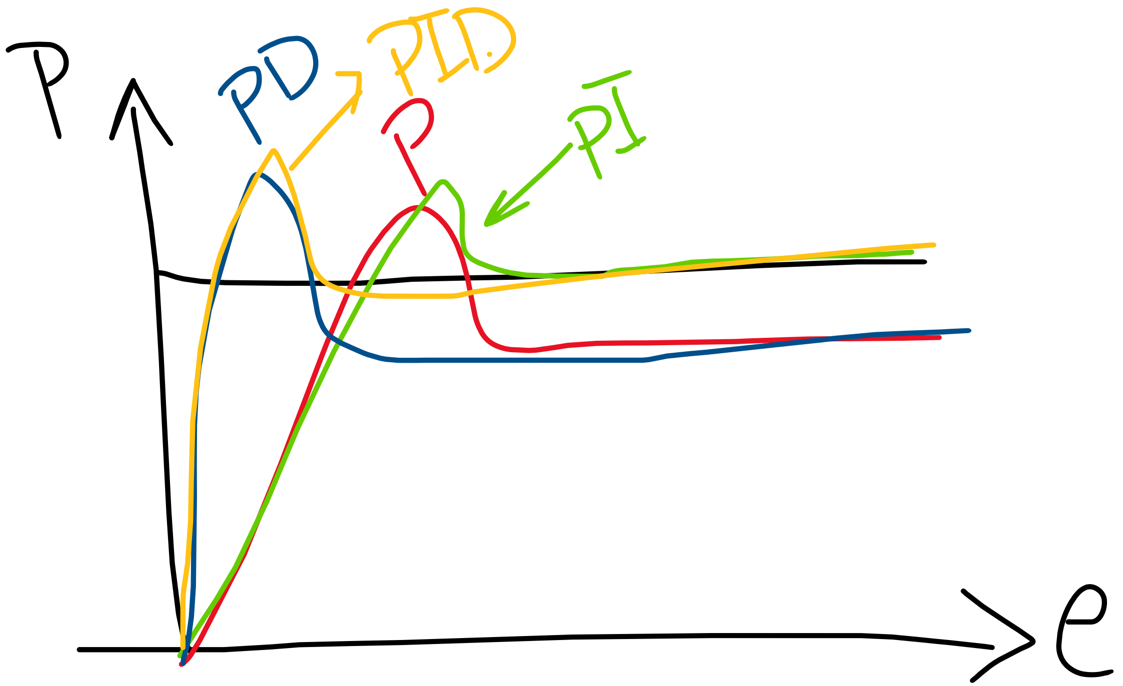
Composite Control Modes:

- Combination of basic modes to gain the advantage of each mode and to eliminate some limitations they individually possess:
  - PI
  - PD
  - PID

Proportional-Integral Control (PI):

$$p = K_P e_p + K_P K_I \int_0^t e_p \, dt + p_I(0)$$

- Adv.: No oscillation, no offset error.
- Dis.adv.: Slow response.

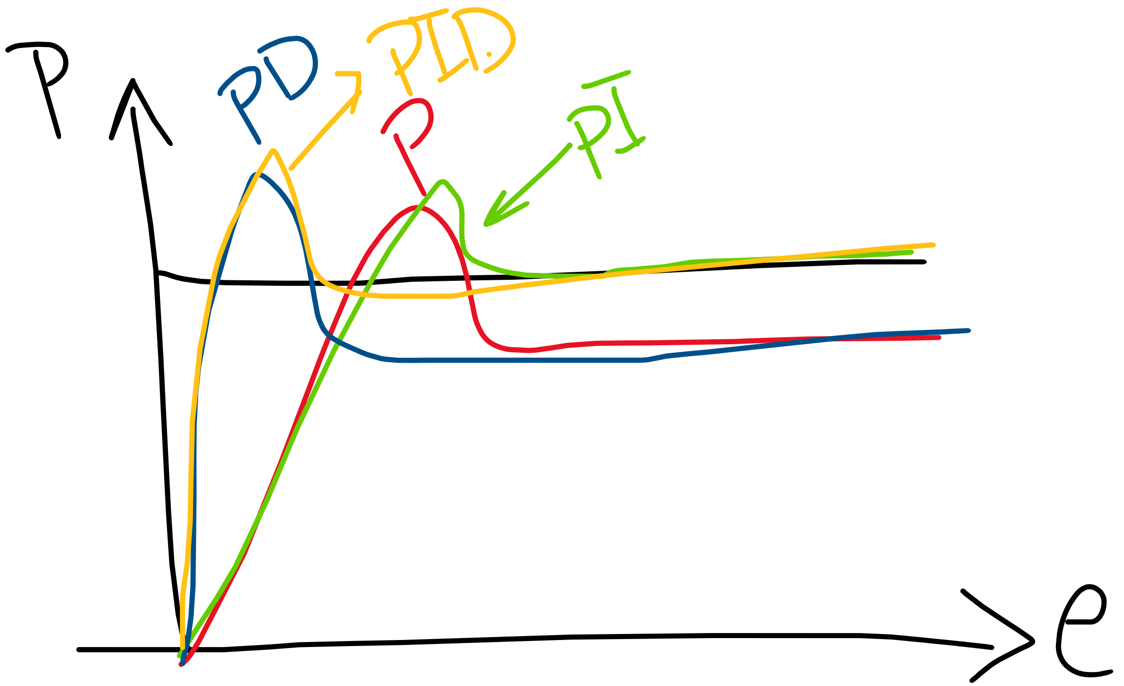


Composite Control Modes:

Proportional-Derivative Control Mode (PD):

$$p = K_P e_p + K_P K_D \frac{de_p}{dt} + p_0$$

- Adv.: No oscillation, fast response.
- Dis.adv.: Offset error.



Three-Mode Controller (PID):

$$p = K_P e_p + K_P K_I \int_0^t e_p dt + K_P K_D \frac{de_p}{dt} + p_I(0)$$

- Adv.: No oscillation, no offset error, fast response.

Composite Control Modes:

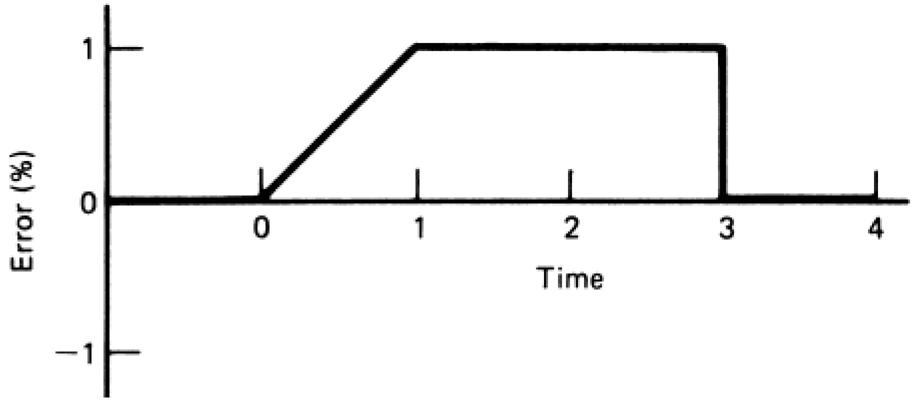
EXAMPLE 8 Given the error of Figure 20 (top), plot a graph of a proportional-integral controller output as a function of time.

$$K_P = 5,\ K_I = 1.0\ \text{s}^{-1},\ \text{and}\ p_I(0) = 20\%$$

Solution

We find the solution by an application of

$$p = K_P e_p + K_P K_I \int_0^t e_p\,dt + p_I(0)$$



$0 \le t \le 1$ $\quad e_p = t$

$$p_1 = 5t + 5\int_0^t t\,dt + 20$$

$$p_1 = 5t + 5\left[\frac{t^2}{2}\right]\Big|_0^t + 20$$

$$p_1 = 5t + 2.5t^2 + 20$$

$$P = \begin{cases} 20 & t = 0 \\ 5 + 2.5 + 20 = 27.5 & t = 1 \end{cases}$$

$$p_1(1) = 2.5t^2 + 20 = 22.5\%.$$

$1 \le t \le 3$ $\quad e_p = 1$

$$p_2 = 5 + 5\int_1^t 1\,dt + 22.5$$

$$p_2 = 5 + 5(t-1) + 22.5 = \begin{cases} 27.5 & t = 1 \\ 37.5 & t = 3 \end{cases}$$

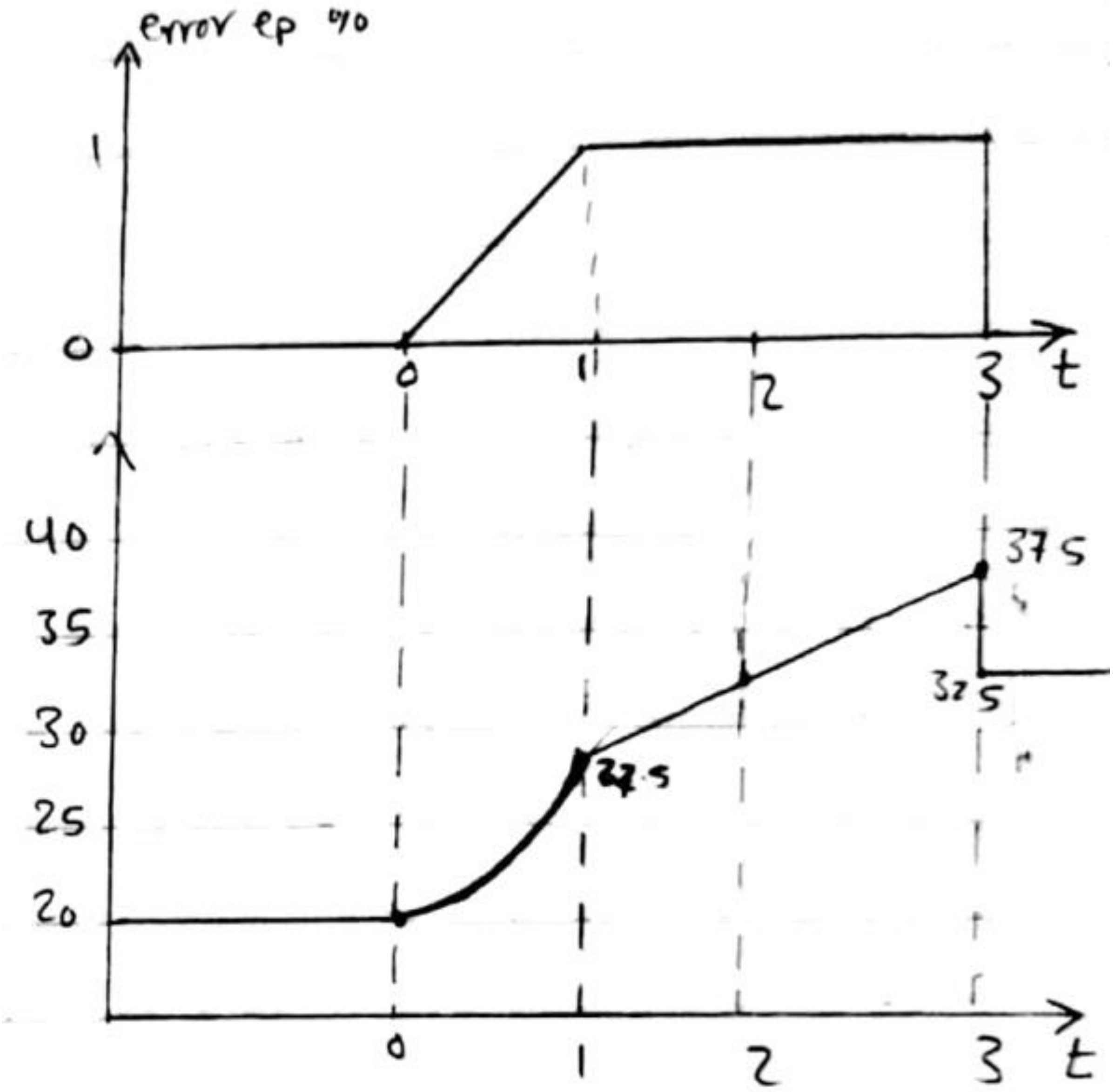
$$p_2(3) = 32.5\%$$

Composite Control Modes:

$t \geq 3 \qquad e_p = 0$

$$p_3 = 5[0] + 5\int_3^t 0\ dt + 32.5$$

$$p_3 = 32.5$$



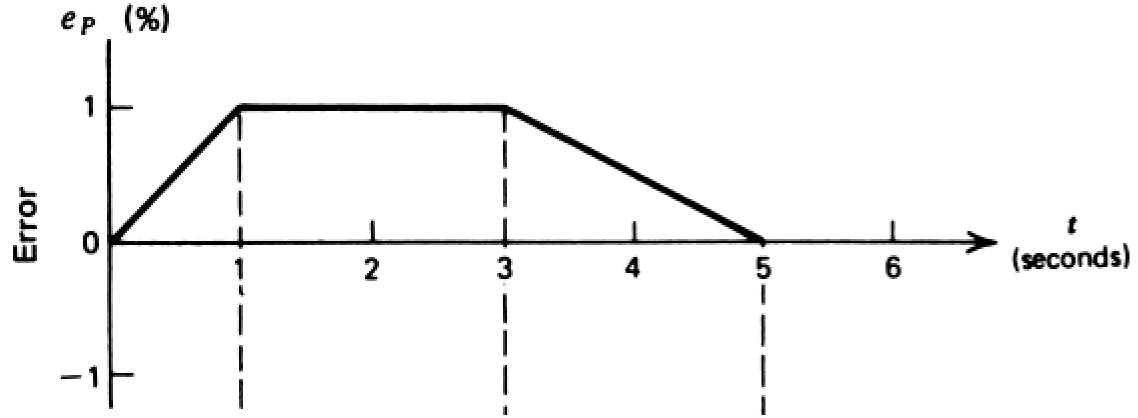
Composite Control Modes:

EXAMPLE 9 Suppose the error, Figure 22a, is applied to a proportional-derivative controller with $K_P = 5$, $K_D = 0.5$ s, and $p_0 = 20\%$. Draw a graph of the resulting controller output.

Solution

In this case, we evaluate

$$p = K_P e_p + K_D K_P \frac{de_P}{dt} + p_0$$



for $0 \leq t \leq 1$: $e_p = t$

$$P = K_p e_p + K_D K_p \frac{de_p}{dt} + P_o$$

$$= 5t + 2.5\,(1) + 20$$

$$P = 5t + 22.5 = \begin{cases} 22.5 & t=0 \\ 27.5 & t=1 \end{cases}$$

for $1 \leq t \leq 3$:

$e_p = 1\%$

$P = 5 + 2.5(0) + 20 = 25\%$

Composite Control Modes:

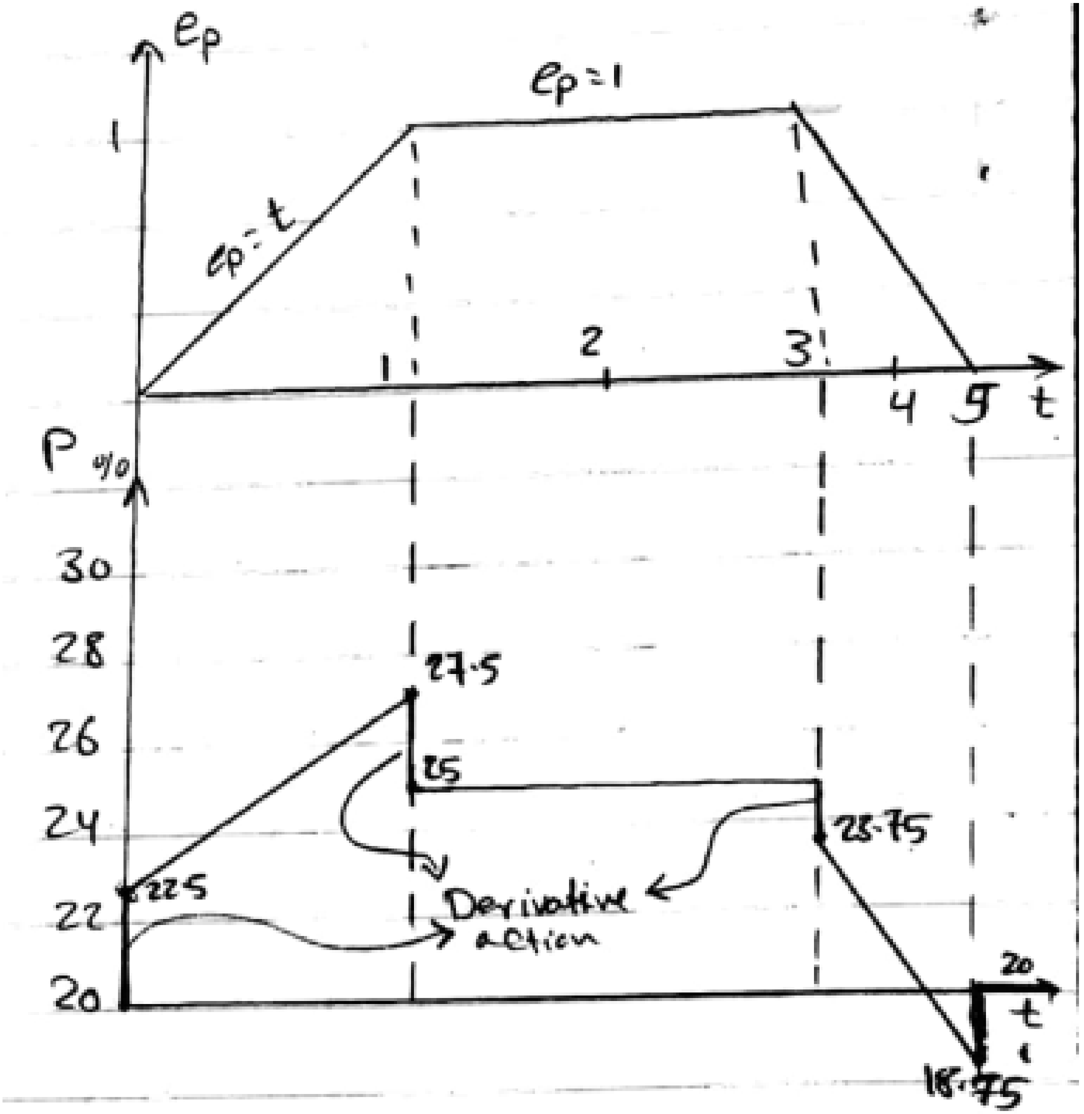
for $3 \leq t \leq 5$

$$\frac{e_p - 0}{t - 5} = \frac{1 - 0}{3 - 5} \Rightarrow e_p = 2.5 - 0.5t$$

$$\therefore P = 5(2.5 - 0.5t) + 2.5(-0.5) + 20$$
$$= 12.5 - 1.25 + 20 - 2.5t = 31.25 - 2.5t = \begin{cases} 8.75 ; t = 3 \\ 18.75 ; t = 5 \end{cases}$$

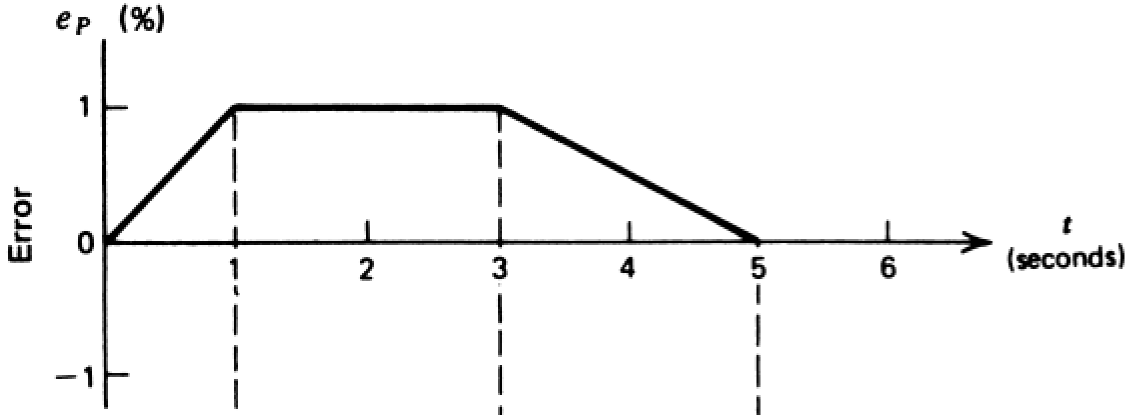
for $t \geq 5$:

$$e_p = 0\,\% \Rightarrow P = P_0 = 20\,\%$$



Composite Control Modes:

EXAMPLE 10 Let us combine everything and see how the error of Figure 22a produces an output in the three-mode controller with $K_P = 5$, $K_I = 0.7\ s^{-1}$, $K_D = 0.5$ s, and $p_I(0) = 20\%$. Draw a plot of the controller output.



Solution

From Figure 22a, the error can be expressed as follows:

0–1 s	$e_p = t\%$
1–3 s	$e_p = 1\%$
3–5 s	$e_p = -\frac{1}{2}t + 2.5\%$

$$p = K_P e_p + K_P K_I \int_0^t e_p\, dt + K_P K_D \frac{de_p}{dt} + p_I(0)$$

for $0 \leq t \leq 1$:

$$P = 5t + \frac{3.5}{2} t^2 \Big|_0^t + 2.5(1) + 20$$

$$P = 1.75t^2 + 5t + 22.5$$

$$= \begin{cases} 22.5 & t=0 \\ 29.25 & t=1 \end{cases}$$

$$P_I(1) = P_I(0) + 1.75t^2 = 20 + 1.75 = 21.75$$

for $1 \leq t < 3$:

$$P = 5 + 3.5 \int_1^t 1\, dt + 2.5(0) + 21.75$$

$$= 26.75 + 3.5(t-1) = 3.5t + 23.25 = \begin{cases} 26.75 \rightarrow t=1 \\ 33.75 \rightarrow t=3 \end{cases}$$

$$P_I(3) = P_I(1) + 3.5 \int_1^3 1\, dt = 7 + 21.75 = 28.75$$

Composite Control Modes:

for $3 \leq t \leq 5$:

$$P = 5(2.5 - 0.5t) + 3.5(2.5t - 0.25t^2)\Big|_3^t + 2.5(-0.5) + 28.75$$

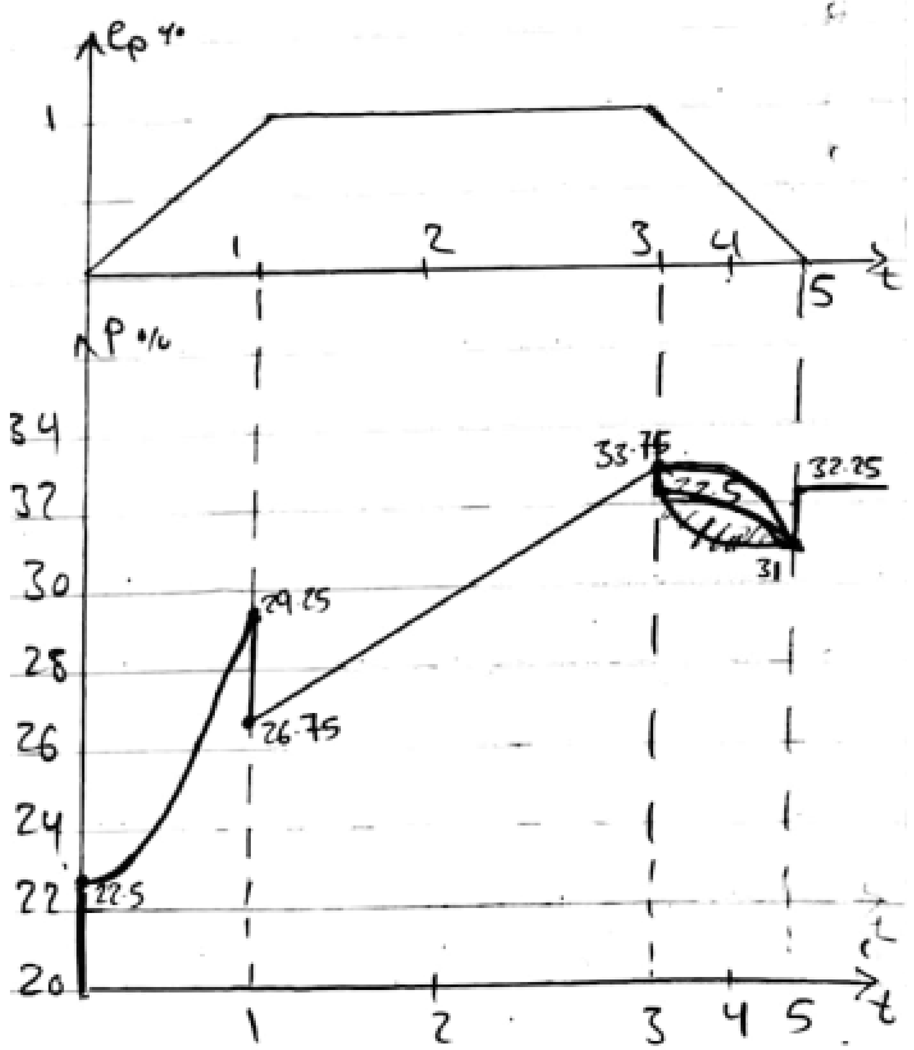
$$P = -0.875t^2 + 6.25t + 21.625$$

$$P = \begin{cases} 32.5 & t = 3 \\ 31\% & t = 5 \end{cases}$$

$$P_I(5) = P_I(3) + 3.5(2.5t - 0.25t^2)\Big|_3^5$$
$$= 28.75 + 3.5$$
$$= 32.25$$

for $t \geq 5$: $e_p = 0$

$$P = P_I(5) = 32.25\ \%$$



END OF LECTURE

BEST WISHES